1. Chapter 3 Problem 5: A binary tree is a rooted tree in which each node has at most two children. Show by induction that in any binary tree the number of nodes with two children is exactly one less than the number of leaves.
2. Chapter 4 Problem 4:

Give an algorithm that takes two sequences of events— *S*’ of length *m* and *S* of length *n*, each possibly containing an event more than once—and decides in time *O(m* + *n)* whether *S*’ is a subsequence of *S*.

1. Chapter 4 Problem 19:

A group of network designers at the communications company CluNet find themselves facing the following problem. They have a connected graph *G* = *(V*, *E)*, in which the nodes represent sites that want to communicate. Each edge *e* is a communication link, with a given available bandwidth *be*.

For each pair of nodes *u*, *v* ∈ *V*, they want to select a single *u*-*v* path *P* on which this pair will communicate. The *bottleneck rate b(P)* of this path *P* is the minimum bandwidth of any edge it contains; that is, *b(P)* = min*e*∈*P be*. The *best achievable bottleneck rate* for the pair *u*, *v* in *G* is simply the maximum, over all *u*-*v* paths *P* in *G*, of the value *b(P)*.

Show that such a tree exists and give an efficient algorithm to find one. That is, give an algorithm constructing a spanning tree *T* in which, for each *u*, *v* ∈ *V*, the bottleneck rate of the *u*-*v* path in *T* is equal to the best achievable bottleneck rate for the pair *u*, *v* in *G*.